

Vuza serialism. Musings on music and mathematics

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ABSTRACT

Who organizes something has to calculate, including one who organizes sound events. Who makes music organizes sound events; so, who makes music has to calculate. Calculating sound is one way of relating to it; but making music means relating to sound in many more ways than just calculating. Hence, there is always a close relation between music and mathematics. But there is also a huge variety of options how important or not mathematical calculus can be in making music. Where one might expect a relation like: the more calculus, the less emotion (or expression), reality proves otherwise – not only in Bach, but he is a prime example. In this essay I would like to venture into the relation between music and mathematics with a practical example of my own – a short composition of mine where I experimented with an extremely constructivist, calculating approach to making music. Starting from a structure of the kind developed by mathematician Dan Tudor Vuza, and used by Violeta Dinescu in one of her recent compositions, my experiment consisted in applying the structure more radically, so as to derive even the pitches and durations from it. This is a somewhat similar step to that from dodecaphonism to serialism; and similar to serialists, I had to ask myself: how far can I go this way, and still make music?

Introduction

This essay will first delve a little into music history in order to give an outline of what serialism is, and how it changed music. Then it will deal with two compositions from the year 2023 in

which something was developed that can, at least ironically, be called Vuza serialism, since it based on a mathematical structure as developed by Dan Tudor Vuza, and in both cases we have to do with extremely constructivist, formalized, mathematic approaches to making music¹.

From tonality to serialism.

“Mode de valeurs et d’intensités”

In the 19th and early 20th centuries, major European composers like Wagner, Scriabin and Stravinsky, Strauss and Schönberg, pushed tonality towards its limits. Schönberg eventually dared to give up tonality altogether. First, he ventured into free atonality, and then he devised a method for atonal composition. It is called the twelve-tone technique since it establishes equality among the twelve half-tone steps inside an octave. None of the pitches is allowed to recur before all the others have appeared. Figure 1 shows an example of a twelve-tone series.

In tonal music, there is a complex hierarchy among the pitches, and hence, among the intervals. This includes various degrees of consonance or dissonance. The intervals regarded as the most consonant are octaves, fifths and thirds, the intervals regarded as the most dissonant are major sevenths, tritones and minor seconds. There is also a dialectics of consonance and dissonance (Figure 2). In the first place, one may think that in tonal music, consonance is seen as something right or desirable, and dissonance as something wrong or undesirable, but this is not really true. One should rather say that, in tonal music, consonance is what marks the *start*

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¹ This essay is based on a presentation given online (via Zoom) in the “Colloquium Arts Meet Sciences” of the European Academy of Sciences and Arts on April 26, 2024 (<https://youtu.be/fsuEGTISxmK>, accessed on March 22, 2025).

or the *arrival* of musical entities, and dissonance is what marks transition.

Let us take a look at a very simple example (Figure 3). The first bar sounds like a beginning, and the last bar sound like an ending, since they are consonant: there are fifths and thirds. The second bar instead sounds neither like a beginning nor like an ending, since it is dissonant: there are major seconds, tritones and major sevenths.

This dialectics of consonance and dissonance is a kind of musical sense-pattern that we understand intuitively. It is kind of second nature to us. We need no technical explanation in order to understand it, it goes by itself. Therefore, it allows of musical communication. Giving up such a kind of sense-pattern means jeopardizing musical communication. So, when Schönberg gives up tonality, when he gives up the hierarchy among the pitches and the intervals, he jeopardizes musical communication. This also concerns traditional musical entities like phrases, themes, complexes of themes, *etc.* Let us take an example as simple and innocent as the melody of “Happy Birthday to You” (Figure 4).

Whoever tries to interpret this melody according to the laws of twelve-tone music will see that a traditional melody like this repeats pitches much more often than twelve-tone music would allow. Played in C major, “Happy Birthday to You” starts on a G. This G is immediately repeated, which is not yet a problem, since even in twelve-tone music, immediate repetition is allowed. But

then, only after one other note, the G recurs; after only two more notes, it recurs again, and so on. In the tonal order, this G has all right in the world to recur so often, since not all tones are equal here, and G has a high rank in this tonal order: It is the dominant, the fifth above the tonic, which is C. But in *twelve-tone atonal* music, all half-tones inside the octave *are equal*. So, if one wanted to make “Happy birthday to You” the theme of a twelve-tone piece, one would at least have to add all of the other half-tones, say, in an accompanying voice, so that G is allowed to recur so often. The result could sound as illustrated in Figure 5.

Now whatever we think of this, if we like it or not, we may agree that it sounds special, extraordinary, that it is an unusual way of arranging this melody. This is something Schönberg or his disciples would not have expected. In fact, they counted on people’s ability of getting accustomed to new sounds; and they had many good reasons to do so. If we look into music history, we find many examples that what we count among the most classical of music was deemed scandalous by its first listeners. This is why Schönberg had many good reasons to count on people’s ability of getting accustomed to new sounds. But in his case, however, it was somehow different. The step from tonality to atonality proved to be somehow more radical than others in the history of music. Even Michael Gielen, one of the greatest musical and musicological promoters of modernism, would admit:



Figure 1. Twelve-tone series.

$$C \xrightarrow{D} C \xrightarrow{D} C \xrightarrow{D} \dots C \xrightarrow{D} C \xrightarrow{D} C$$

Figure 2. Scheme for the function of consonance and dissonance in tonal music.

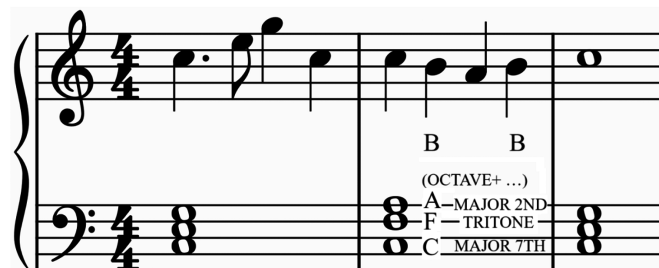


Figure 3. Example for the function of consonance and dissonance in tonal music.



Figure 4. The beginning of “Happy Birthday to You” in C major.



Figure 5. The beginning of “Happy Birthday to You” in C major, yet arranged dodecachonically.

This is the enormous difficulty of modern music, i.e., the difficulty Schönberg faced after abandoning tonality, that the particles that themes or thematic structures are composed of are hard to remember. Schönberg hoped that one day people would whistle his music in the street, but hardly anyone listens to it often enough to be able to hum along. In this respect, I think it's fair to say that tonality, the organization of large forms through the tonal functions of triads in a key, is an element that constitutes meaning, an element of communication between creators and listeners, that is, between composers and audiences, whose loss cannot be lamented loudly enough. It is not the only thing that has been lost, but from a musical and technical perspective, the thread of communication between creator and audience was practically broken the moment tonality had to be abandoned for historical reasons. [...] Articulation of form through musical elements (such as the harmonic scale degrees of tonality) must be presupposed there, but one cannot really retrace its logic while listening².

After World War II, the Darmstädter Ferienkurse für Neue Musik (Darmstadt Summer Courses for New Music) became an international hub for music history. Schönberg's twelve-tone technique not only saw a renaissance there, but a radicalization. In 1949, great French composer Olivier Messiaen made an experiment: He wrote a piano piece, called “Mode de valeurs et d'intensités (Mode of durations and intensities)”. In this piece, all of the pitches, durations, intensities and kinds of attack are regulated by an inventory, called mode. This mode was similar enough to a series à la Schönberg that many young composers, like Pierre Boulez, Karlheinz Stockhausen and Bernd Alois Zimmermann, saw the future of music in not only organizing pitches according to series, but also other parameters like duration, intensity and sound color. This is definitely no longer a music in which one could quote “Happy Birthday to You”. In Schönberg's twelve-tone technique, this was still possible, even if it came at a high price. The same price now comes to the power of four.

Therefore, at least in its raw state, the serialist method cut music off from everything that had traditionally been called music. Why were the young post-war composers willing to make such a radical step? – One of the motifs seems to have been that being truly modern included doing science, and that truly modern art was supposed to be of scientific spirit. When the creative process was reduced to the determination of series from which everything else would be mathematically induced, this seemed to be the ultimately modern way of making music. One needs to ask, of course, if human creativity thereby does not become entirely insignificant, such that computers, and not humans, would be the

ideal composers and interpreters of such perfectly modern music. On May 5, 2023, Prof. Klaus Mainzer spoke to us about the challenges posed by AI to the concept of human creativity³. In a certain sense, about 70 years ago, serialism at least came close to defining AI as the perfectly modern kind of creativity.

Now one needs to stress, in defense of the great serialists, that they never allowed it to come that far. Composers like Boulez, Stockhausen and Zimmermann found ways of giving flexibility to their method. Yet they kept struggling with the difficulties of making their music understandable, of restoring musical communication. Especially Zimmermann despised any technicism and constructivism, just as Schönberg did. To them, music was still essentially expression of the human mind and soul, and hence dependent on something like inspiration, not on calculus alone⁴. Like its composition, its interpretation was supposed to be done not by computers, but by living, breathing, moving, feeling, thinking human beings. As impressed and convinced as Zimmermann was of serialism in general, whenever calculus threatened to *hinder* true human expression, it was to be ignored.

² My translation; in the German original: “Das ist ja die ganz große Schwierigkeit der moderneren Musik, also die Schwierigkeit Schönbergs nach Aufgabe der Tonalität, daß die Partikel, aus denen sich Themen oder thematische Gebilde zusammensetzen, schwer zu erinnern sind. Schönberg hat gehofft, man würde seine Musik eines Tages auf der Straße pfeifen, aber kaum einer hört sie sich oft genug an, um sie auch nur mitsummen zu können. Da darf man, meine ich, auch sagen, daß die Tonalität, die Organisation großer Formen durch die tonalen Funktionen der Dreiklänge in einer Tonart, ein sinnstiftendes Element ist, ein Element der Kommunikation zwischen den Schaffenden und den Rezipierenden, also zwischen Komponisten und Publikum, dessen Verlust gar nicht laut genug zu beklagen ist. Es ist nicht das Einzige, was verloren gegangen ist, aber musikalisch-technisch ist der Faden der Kommunikation von Schöpfer und Publikum praktisch in dem Moment abgerissen, da die Tonalität, aus geschichtlichen Gründen, aufgegeben werden mußte. [...] Artikulation von Form durch musikalische Elemente (wie sie die harmonischen Stufen der Tonalität waren) muß man da voraussetzen, wirklich nachvollziehen kann man sie nicht”. Gielen M, Fiebig P (2002) pp. 151 sq.

³ The title of the presentation was “Digital Intelligence – Challenge for Human Creativity?” (<https://youtu.be/iux31h2dsas>, accessed on March 22, 2025).

⁴ As early as 1949, Zimmermann reminded his Darmstadt colleagues that “making twelve-tone music naively according to the recipe” was “something that Schönberg protested against categorically” – my translation; in the German original: “daß man kreuzfidel nach rezeptmäßiger Anleitung Zwölftonmusik macht, ein Vorgang, gegen den Schönberg mit aller Entschiedenheit protestiert hat” (Zimmermann BA, Technik und Inspiration, in: Borio G, Danuser H, 1997; pp. 469-471, here p. 471).

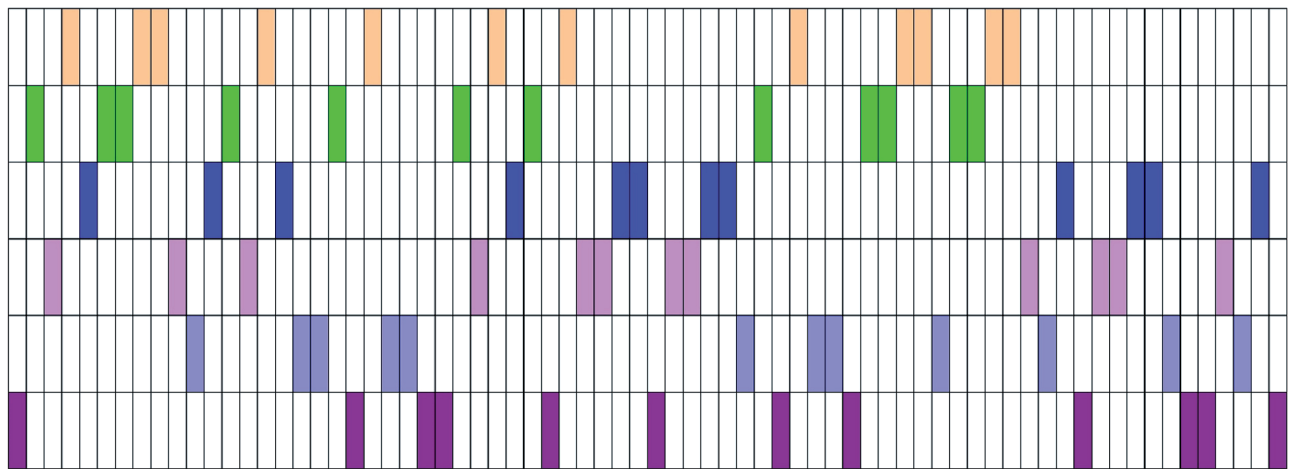


Figure 6. Vuza structure of Violeta Dinescu's composition "Cronicari", taken from the work's manuscript. A recording of the first performance can be found here: <https://youtu.be/1ugbFdW3mKY>

Violeta Dinescu's Vuza canons to Vuza serialism. "Mode de valeurs et d'hauteurs"

Now I must admit that whenever I compose something, my standard procedure is having some musical idea that convinces me, and then developing its inner logic. That is, I start with inspiration, not with calculus. First there must be a musical idea that, for whatever reason, convinces me. Calculus comes with the development of this idea's inner logic. But I see that there are other composers who do not proceed this way, and that even I myself sometimes do it differently. There is extremely expressive music whose purpose, in the first place, was not expression. Bach's "Art of the Fugue" is an extreme example.

In 2023, Violeta Dinescu wrote a piece called "Cronicari (The Chroniclers)", in which she uses a structure as developed by Romanian mathematician Dan Tudor Vuza in a series of articles, starting with "Sur le rythme périodique (On the periodic rhythm)" in 1985 (Vuza, 1985; 1991; 1992a; 1992b; 1993; 1999; 2024). The structure used by Dinescu is illustrated in Figure 6. The structure has six horizontal lines and 72 vertical lines. It would be more adequately represented not, like here, as a flat ribbon, but as a cylinder. One just needs to imagine that the short ends of the ribbon are glued together.

Now what makes this structure a Vuza structure? – It is that in each horizontal line, we have a non-periodical pattern of blank and full spaces, that is, of "0s" and "1s"; the number of "1s" relates to the number of horizontal lines as multiplied or divided by a natural number; and although the colors in this illustration are different, the pattern is one and the same on each horizontal line. One can see it particularly well if one compares horizontal lines 1 and 2, or 3 and 4: it is always the same pattern, but from line to line, this pattern is shifted against itself, and it is shifted in such a way that no vertical line has more than one "1".

A particular feature of such structures is that the number of their horizontal lines must at least have two prime factors. Therefore, there can be none with less than six horizontal lines; the Vuza structure shown in this example corresponds to this minimum demand.

A structure like this can be given a musical interpretation by

taking the vertical dimension as a set of voices, by taking the horizontal dimension as the timeline, and by taking the "1s" as sonic moments⁵. There are various ways how a voice may be defined as such, how the time is measured, and how the sonic moments can be determined.

For example, the 72 vertical lines are interpreted as 72 time slots for the voices. The "1" on each vertical line then indicates whose turn it is. For example, at the very beginning, voice 6, then voice 2, etc. But since there is never more than one "1" on each vertical line, the voices never overlap (Figure 7).

The musical result is a particular kind of canon. In a canon, several voices follow each other in an identical way. In a traditional canon, the voices play the same thing at different entries, so that the voices overlap. But here, the voices follow each other in another way. They do not play the same thing at different entries, but they have the same sequence of sonic moments – taken to be time slots – with "0s" and "1s" at different entries. The result is something that first appears like a contradiction in itself: Normally, a canon is understood as a composition with several voices overlapping in playing the same thing; but a Vuza canon where the sonic moments are taken to be time slots is a canon in which several voices neither play the same thing, nor overlap.

Vuza himself would explain this in different terms, namely as one voice establishing the inner rhythm of the canon and the other voices relating to it as the outer rhythm. Concerning the original mathematical conception, the reconstruction given here is admittedly simplistic; but it is sufficient within the scope of what is to be shown here.

As said above, the sonic moments determined by the Vuza structure need not be interpreted as time slots for the voices. They can also, for example, be taken to determine attacks or accents, such that the voices can overlap, though their attacks, respectively accents, do not.

Prior to the composition "Cronicari", Dinescu composed several Vuza canons, with no overlapping voices. In "Cronicari", the first part is a strict Vuza canon, without overlapping voices. The

⁵ Among the composers who – apart from Violeta Dinescu – have done this so far are Laura Manolache, Fabien Lévy and Mauro Lanza.

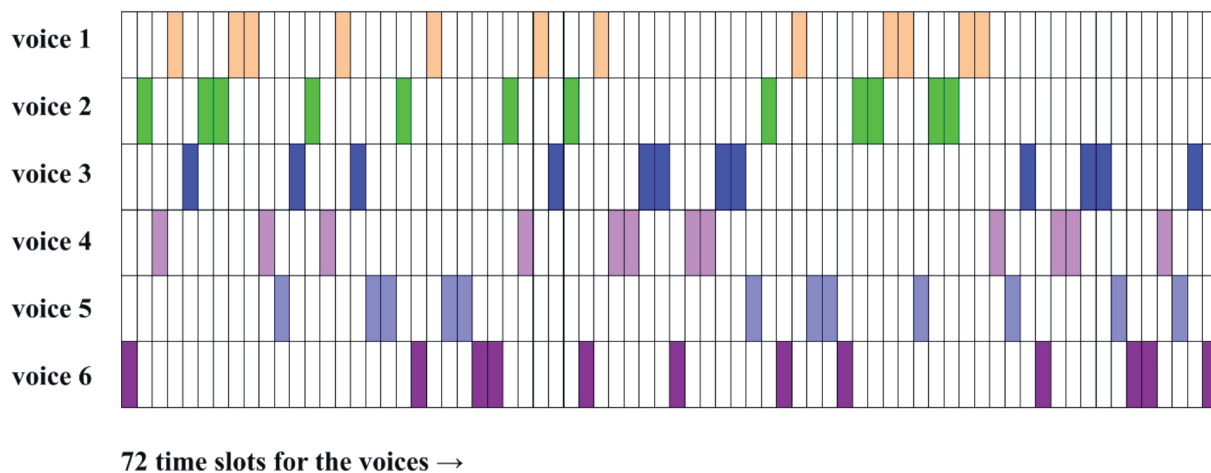


Figure 7. The Vuzza structure and its interpretation as a canon in the first part of Violeta Dinescu's "Cronicari".

second part is a Vuzza canon of accents, such that the voices, yet not the accents, are allowed to overlap. The third part follows the same principle as the second, but with the addition of a supplementary voice (flute and spoken text). The voices follow the rhythmic structure of the Vuzza canon, but each voice contains different musical material.

Such a manner of applying the Vuzza structure musically does not prescribe in any way what *kind* of music is to be played, but only when which voice is active, or attacks, or makes an accent, and the others not. This made me think if there could be a way also to derive the musical *material* from the structure. Indeed, there was an easy way for doing so, since 72 is a multiple of 12, hence of the number of half-tones of the chromatic scale. I could take the Vuzza structure and divide it into six sections, each comprising 12 vertical lines. Each of the 12 vertical lines then was interpreted as one step on the chromatic scale from C to B, and as a sixteenth. If, for example, as at the beginning of voice 1, the Vuzza structure section is 000100011000, we get the following pitches: 4=D#, 8=G, and 9=G#; and we get the following durations: a pause of a dotted eighth, a quarter, a sixteenth and another quarter.

This is how I made the structure determine the musical mate-

rial concerning pitches and durations, which always stay strictly coupled (Figure 8). The portioning of the structure made the single voices change each time after 12 vertical lines, that is, after 12 time slots. I liked this variety, but since the twelve-tone series I had applied was a mere ascending chromatic scale, the intervals in each bar moved uniformly upward (of course I could have chosen a more sophisticated twelve-tone series, but I did not want the whole thing to become all too intransparent as to its Vuzza origins).

Now the question of questions was: How would it sound like? – With an allegro tempo, the first result of the procedure sounded neither horrible nor overwhelming. It sounded definitely very different from serialist music, it allowed of pitch and duration repetitions and of motif repetitions, it also allowed of tonal effects without being tonal at all. In spite of its repetitiveness there was enough variety and syncopation to prevent it from being boring. It appeared to me that, if played much more slowly and on the clarinet, it could have a solemn, conjuring expression. This made me decide that the *tempo* would *not* be determined by the structure. If this acoustic derivative of mathematics could be received as expressive, hence as music, this chance should not be missed by overdoing mathematics.

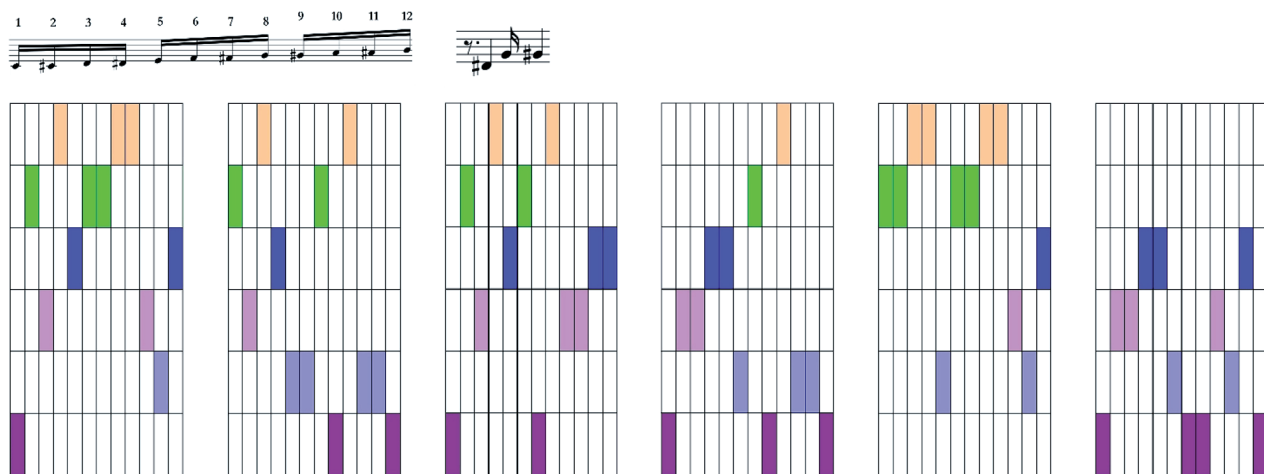


Figure 8. The Vuzza structure of "Cronicari" as used by myself.



Figure 9. Detail of the new composition before (left) and after (right) the adjustment of the octaves.



Figure 10. The same detail of the new composition.

The problem of the uniform upward movement was solved together with another one: on the clarinet, I have three well-sounding octaves, and how should I distribute the pitches to the octaves? This was determined by the first line of the Vuza structure, applied to the whole of the piece (Figure 9). Whenever there is a “0” in the Vuza structure, each sound is to be played on the lowest octave possible; when there is a “1” preceded by a “0”, the middle octave; and when there is a “1” preceded by a “1”, the highest. Since the octaves go from E to E, and not from C to C, that gives the intervallic movement more variety⁶.

When I tried it out on the clarinet, I took one last basic decision: namely, that also the *dynamics* would *not* be determined by a structure that does not know anything of breath-lengths, of dynamic tensions and developments, of phrasing, of expression, of music altogether. As I said before: If this purely number-born piece could have a solemn, conjuring expression, the numbers should not hinder it. A sixteenth at the end of one bar, directly followed by a sound in the next bar, would be received by any listener as an anacrusis, even though it is derived from a structure that does not know anything of what an anacrusis is (Figure 10).

I could have distinguished the voices by different playing techniques, but I chose not to, since even in the standard technique, sound color changes across the registers, and the extended techniques are even less indifferent concerning pitch, duration, and intensity. During performance I also did not stick so strictly to the metronome mark - which encodes a detail of the work’s occasion - otherwise, the piece would be one minute shorter⁷. Hopefully, the rest is music! So here is the first and maybe also last example of Vuza serialism in music history (Figure 11). It shall be called “Mode d’hauteurs et de valeurs (Mode of pitches and durations)” since it derives these musical

parameters out of a technique that has already been used in music, but not like this, somewhat akin to what the serialists did with dodecaphonism, inspired by Messiaen’s “Mode de valeurs et d’intensités (Mode of durations and intensities)”.



Figure 11. My composition “Mode d’hauteurs et de valeurs” for solo clarinet, composed in 2023.

⁶ Thereby I stuck to the intervallic values as they are *notated* – that is, I ignored the fact that any pitch played by the B-flat clarinet sounds a whole tone lower, such that, as the intervallic values *sound*, any octave here goes from D to D.

⁷ A recording of the piece can be found here: <https://youtu.be/7xxOYOskeyo> (accessed on March 23, 2025).

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